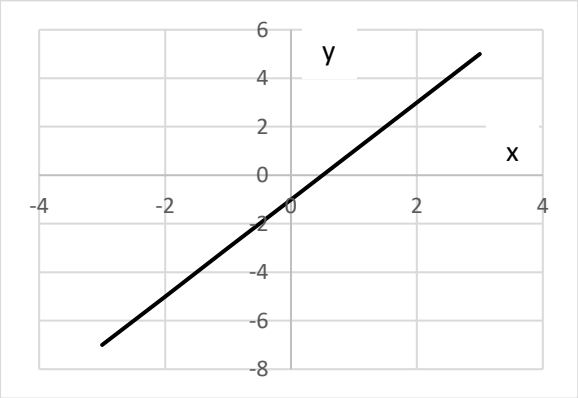
Functions 2 – The Modulus Function

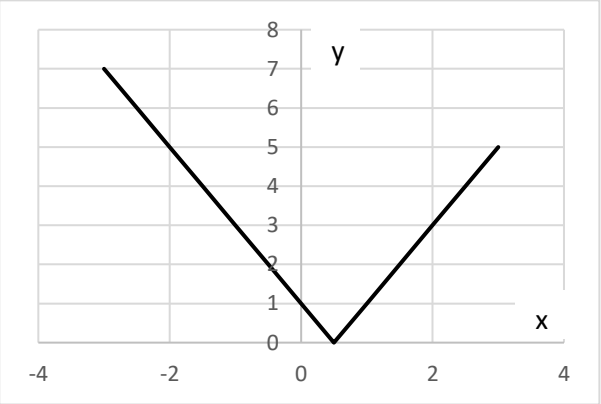
The Modulus Function and its Graph

The modulus function returns the positive value of its argument. For example, $|-3| = 3$. Also, it is often used for statements about when a series converges, e.g. the series might be said to converge when $|x| < 1$, i.e., all values of x in the range $-1 < x < 1$. Its use can be extended to apply to general functions. The graphs below are plots of the three functions $f(x) = (2x - 1)$, $|f(x)| = |2x - 1|$ and $f(|x|)$.

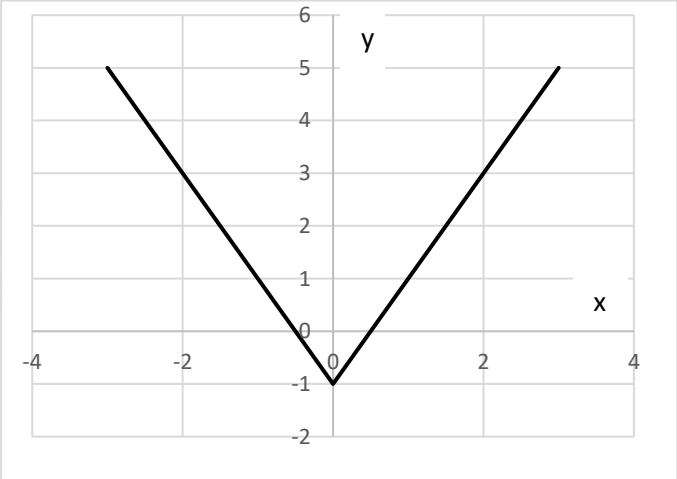
$$y = f(x) = (2x - 1)$$



$$y = |f(x)| = |2x - 1|$$



$$y = f(|x|) = 2|x| - 1$$

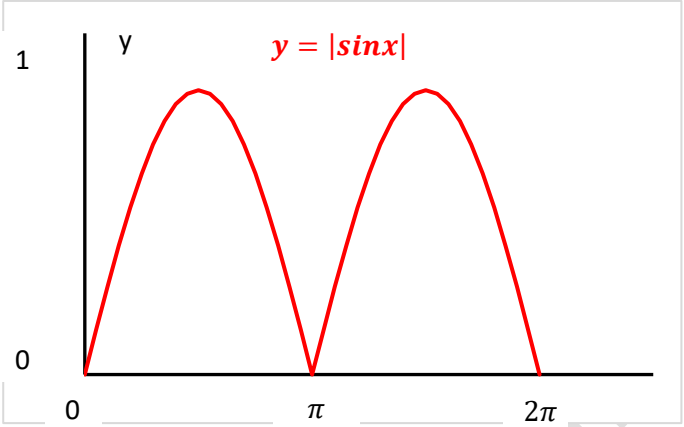


Points to note

1. The original function $f(x) = 2x - 1$ is a straight line and can take both positive and negative values.
2. The second function $|f(x)| = |2x - 1|$ can only take positive values. $f(x) = 0$ when $x = \frac{1}{2}$ and so $|f(x)|$, is a reflection of the curve in the line $x = \frac{1}{2}$.
3. Since $|x|$ is the absolute value of x the function $f(|x|)$, is a reflection of the function in the line $x = 0$, i.e., in the y-axis. (The function can go negative as shown above.)

Note

For the plot of the $|sinx|$ function, the act of taking the modulus has resulted in the negative portion of the curve being reflected in the x-axis to become positive.



It is important to be able to visualise these types of graphs if you need to solve, for example, equations of the form $y_1 = |2x - 1|$ and $y_2 = x$ simultaneously. The solution(s) are the points of intersection of the two graphs.