Algebra2:- Completing the square and roots of quadratics

Completing the square

This is just another way of writing the quadratic

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a}\right) \quad \text{for } a \neq 0$$

Multiply the brackets out and collect terms to check that it is correct.

If $a > 0$, $\quad a\left(x + \frac{b}{2a}\right)^2 \geq 0$ and so we can conclude the minimum value of the quadratic is $-\left(\frac{b^2-4ac}{4a}\right)$ and it occurs when $x = -\frac{b}{2a}$.

If $a < 0$, $\quad a\left(x + \frac{b}{2a}\right)^2 \leq 0$ and so we can conclude the maximum value is $-\left(\frac{b^2-4ac}{4a}\right)$ and it occurs when $x = -\frac{b}{2a}$.

Example: $2x^2 - 8x + 9 = 2(x^2 - 4x) + 9$

$$= 2[(x-2)^2 - 4] + 9$$

$$= 2(x-2)^2 + 1$$

This has a minimum value of 1 at $x = 2$.

Roots of a quadratic

The roots of the equation $ax^2 + bx + c = 0$ are those of $a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2-4ac}{4a}\right) = 0$

i.e., $\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b^2-4ac}{4a^2}\right)$ which are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

The discriminant is $b^2 - 4ac$.

If $b^2 - 4ac > 0$ there are two distinct real roots.

If $b^2 - 4ac = 0$ there is one real root (some would say one repeated root)

If $b^2 - 4ac < 0$ there are no real roots.

If the quadratic factorises then you don't need the above formula, although it will still work.

The following are all quadratic forms.

Quadratic in $tanx$

$2tan^2x + 3tanx + 1 = 0.$ Factorising gives $(2tanx + 1)(tanx + 1) = 0$

Roots are $tanx = -\frac{1}{2}$ and $tanx = -1$

Quadratic in e^x $3e^{2x} - 7e^x + 2 = 0.$ Factorising gives $(3e^x - 1)(e^x - 2) = 0$

Roots are $e^x = \frac{1}{3}$ $i.e., \ x = -ln3$ or $e^x = 2$ $i.e., \ x = ln2$