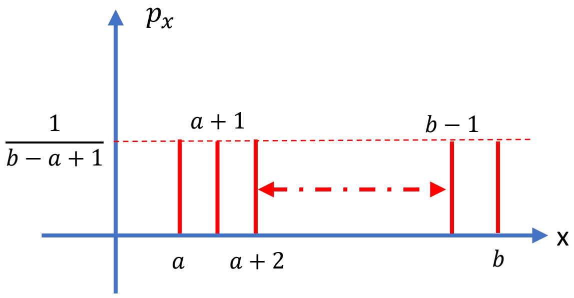
Specific Discrete distributions

Discrete Uniform Distribution

Here we have a sample space of the form $a, a+1, a+2, \dots \dots, b-1, b$, where there are $(b-a+1)$ values, each with probability $\frac{1}{(b-a+1)}$. The fact that this probability is the same for each value of x, gives rise to the name "uniform distribution". It is displayed in the figure alongside.



The mean of a uniform discrete distribution $E(X) = \sum_a^b x p_x = \frac{1}{(b-a+1)} \sum_a^b x = \frac{(a+b)}{2}$

The variance of this uniform distribution is $Var(X) = E(X^2) - E^2(X) = \frac{(b-a+1)^2 - 1}{12}$

Binomial distribution

Here we consider a process that has **n independent trials** with only two outcomes per trial, success and failure say. Also, it is assumed that the **probability of a success is p**, which is the **same for each trial**.

For convenience let the outcomes for each trial be denoted by S(success) and F(failure) respectively. The first few probability distributions for small n are.

n=1 Sample space $= \{F, S\}$ Probability distribution:-

$$P(S=0) = (1-p); \quad P(S=1) = p$$

n=2 Sample space $= \{FF, FS, SF, SS\}$ Probability distribution

$$P(S=0) = (1-p)^2; \quad P(S=1) = 2(1-p)p; P(S=2) = p^2$$

n=3 Sample space $= \{FFF, FFS, FSF, SFF, SSF, SFS, FSS, SSS\}$ Probability distribution

$$P(S=0) = (1-p)^3; \quad P(S=1) = 3(1-p)^2 p; \quad P(S=2) = 3(1-p)p^2; \quad P(S=3) = p^3$$

As can be seen the constant coefficients for n=2 are 1, 2, 1 and for n=3 they are 1, 3, 3, 1. These are the binomial coefficients, and hence the name of the distribution.

Generally, if there are n trials and x successes the probability $P(S=x) = C_x^n p^x (1-p)^{n-x}$. There are x successes with probability p^x, $(n-x)$ failures with probability $(1-p)^{n-x}$ and there are C_x^n ways of arranging x successes and $(n-x)$ failures.

Note:- For the binomial distribution to be appropriate you must have n independent trials with constant probability of success for each trial. Also the notation usually used to denote this distribution is **B(n,p), B for Binomial, n for the number of trials, and p for the probability of a trial success.**
If the random variable X has a B(n,p) distribution it can be shown that

$$Mean = E(X) = \sum_{x=0}^{n} x C_x^n p^x (1-p)^{n-x} = np$$

$$Var(X) = E(X^2) - E^2(X) = np(1-p)$$

Note:- Probabilities and cumulative probabilities can be found in tables or on modern calculators.

Example: Suppose the random variable X has a B(10,0.45) distribution. Then

$$F(6) = P(X \le 6) = 0.8980; \quad F(7) = P(X \le 7) = 0.9726$$

Therefore $\quad P(X = 7) = F(7) - F(6) = 0.9726 - 0.8980 = 0.0746$

Below are two binomial distributions B(20,0.05) and B(40,0.25). They show the discrete nature of the distributions and also that the binomial can be very skewed if p is small and n not too large, although when n is moderately large the shape of the binomial becomes more symmetric. We will return to this shortly.